SUB.CODE: 18PMM4C11

REG.NO:

DHANALAKSHMI SRINIVASAN COLLEGE OF ARTS & SCIENCE FOR WOMEN (AUTONOMOUS)

(For Candidates admitted from 2019-2020 onwards)

PG DEGREE EXAMINATIONS APRIL – 2021

M.Sc., - MATHEMATICS

DIFFERENTIAL GEOMETRY

Time: 3 Hrs **Max.Marks: 75**

PART - A

CHOOSE THE CORRECT ANSWER **(10X1=10)**

1. The centre of the osculating radius is the radius of __________ curvature
   a) curve b) sphere c) point d) spherical
2. For a helix K/J is equal to ________
   a) 1 b) constant c) 0 d) 5
3. A point which is not an ordinary point s is ____________
   a) ordinary b) singular c) one point d) none
4. Every family of curves on a surface possesses a family of trajectories _______
   a) orthogonal b) ortho-normal c) normal d) curves
5. A surface of revolution which is isometric with the right ________
   a) surface b) helicoid c) curves d) isometric
6. Equation of the tangent plane to the surface $z = xy$ at the point (2,3,6) is _________
   a) $3x-2y-z=6$ b) $3x - 2y + z = 6$ c) $3x+2y-z=6$ d) none
7. A region R on a surface is ___________ if any two points of the surface can be joined by a geodesic arc
   a) concave b) convex c) normal d) parallel
8. Plane of curvature is called _________
   a) rectifying plane b) normal plane c) osculating plane d) none
9. The values of the Gaussian curvature is ___________ of the parametric system
   a) dependent b) circle c) curves d) independent
10. The envelope of the normal plans to the space curve is the ________ developable
    a) polar b) Cartesian c) curves d) surface

PART- B

ANSWER ALL THE QUESTIONS **(5X7=35)**

11. a) Show that $[r',r'',r'''] = 0$ is a necessary and sufficient condition that the curve be plane.

(OR)

b) Prove that the curve $x = au$, $y = bu^2$, $z = cu^3$ is a helix if $3ac = \pm 2b^2$

12. a) Prove that the position vector of a point on the anchor ring is given by r = [(b + a cos u) cos v, (b + a cos u) sin v, a sin u)] where (b, 0, 0) is the centre of the circle and z-axis is the axis of rotation.

(OR)

b) If ω is the angle between the parametric curves at the point of intersection, the prove that $\tan \omega = \frac{H}{F}$

13. a) Find the surface of revolution which is isometric with the region of the right helicoid.

(OR)

b) If θ is the angle between the two curves double family $P\,du^2 + 2Q\,du\,dv + R\,dv^2$, at a point (u, v) on the surfaces, then prove that

$$\tan \theta = \frac{2H\left(Q^2 - PR\right)^{1/2}}{ER - 2FQ + GP}$$

14. a) For any given family of geodesic on a surface then prove that a parametric system can be chosen so that the metric takes the form $ds^2 = du^2 + G(u, v)\,dv^2$

(OR)

b) For any curve on a surface prove that the geodesic curvature vector is intrinsic.

15. a) Prove that a necessary and sufficient condition for a curve on a surface to be a line of curvature is κ **dr** + d**N** = 0 at each point of the line of curvature where κ is the normal curvature in the direction dr of the line of curvature

(OR)

b) Show that the necessary and sufficient condition for a surface to be a developable is that its Gaussian curvature is zero.

PART-C

ANSWER ANY THREE QUESTIONS **(3X10=30)**

16. Find the equation of curve whose curvature and torsion are constants
17. The first fundamental form of a surface is a positive definite quadratic form in du, dv
18. State and prove Gauss-Bonnet theorem
19. Any curve u = u(t), v = v(t) on a surface r = r(u, v) is a geodesics iff the principal normal at every point on the curve is normal to the surface.
20. Show that the principal directions are given by the roots of the equation

$$(\kappa^2 (EG - F^2) - \kappa (EN + GL - 2FM) + LN - M^2 = 0$$