SUB.CODE: 18PMM4C13

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DHANALAKSHMI SRINIVASAN COLLEGE

OF ARTS & SCIENCE FOR WOMEN

(AUTONOMOUS)

(For Candidates admitted from 2019-2020 onwards)

PG DEGREE EXAMINATIONS APRIL – 2021

M.SC – MATHEMATICS

MODERN PROBABILITY THEORY

Time: 3 Hrs **Max.Marks: 75**

PART A

CHOOSE THE CORRECT ANSWER. $(10 \times 1 = 10)$

1. If A and B are independent events then $P(A \cap B) =$ ______________.

   (a) $P(\bar{A}) . P(\bar{B})$ (b) $P(A) - P(B)$ (c) $P(A) + P(B)$ (d) $P(A) . P(B)$

2. One of the properties of Distribution function is __________.

   a) $F(x) \leq -1$ b) $0 < F(x) < 1$ c) $0 \leq F(x) \leq 1$ d) $F(x) \geq 1$

3. If X and Y are two independent random variables then $E(XY) =$ ____________.

   a) $E(X)|E(Y)$ b) $E(X) . E(Y)$ c) $E\big(X - E(Y)\big)^2$ d) $E(X) + E(Y)$

4. $E(X^2) - \big(E(X)\big)^2 =$ ____________.

   a) $E\big(X - E(Y)\big)^2$ b) $E\big(X + E(Y)\big)^2$ c) $E\big(X \pm E(Y)\big)^2$ d) $[E\big(X - E(Y)\big)]^2$

5. For continuous probability distribution $E\big(e^{itx}\big) =$ ___________.

   a) $\int_{-1}^{1} e^{itx} . f(x)dx$ b) $\int e^{itx} . f(x)dx$ c) $\int e^{itx} . dx$ d) $\int_{-\infty}^{\infty} e^{itx} . f(x)dx$

6. The function defined as $\Psi(s) = E(s^x)$, where $|s|$___________.

   a) ≥ 1 b) ≤ -1 c) ≤ 1 d) ≥ -1

7. If $\varphi(t) = e^{itm - \frac{t^2\sigma^2}{2}}$ then the characteristic function of ____________ Distribution.

   a) Hyper geometric b) Beta c) Gamma d) Normal

8. The characteristic function of Gamma Distribution is _______________.

   a) $\left(1 - \frac{it}{a}\right)^{-p}$ b) $\left(1 + \frac{it}{a}\right)^{-p}$ c) $\left(1 - \frac{it}{a}\right)^{p}$ d) $\left(1 + \frac{it}{a}\right)^{p}$

9. If $\{A_n\}$ is an independent sequence of events such that $\sum_{n=1}^{\infty} P(A_n) = \infty$ then $P(A) =$ ________.

   a) ∞ b) 1 c) 0 d) -1

10. If $X_n \xrightarrow{P} X$ is convergence in probability then for $\in > 0$, as $n \to \infty$,____________.

a) $P[|X_n - X| \leq \in] \to 0$

b) $P[|X_n + X| \geq \in] \to 0$

c) $P[|X_n - X| \geq \in] \to 0$

d) $P[|X_n - X| \geq \in] \to \infty$

PART – B

ANSWER ALL THE QUESTIONS. **$(5 \times 7 = 35)$**

11. a) State and prove the Bayes theorem.

(OR)

b) There are four slips of paper of identical size in an Urn. Each slip is marked with one of the numbers $110, 101, 011, 000$, and there are no two slips marked with same number. Show that pairs are independent.

12. a) State and prove the chebychev's inequality.

(OR)

b) Find the regression curves for the two dimensional normal distribution?

13. a) The characteristic function of the random variable x is $\varphi(t) = \exp\left(-\frac{t^2}{2}\right)$. Find the density function of random variable x?

(OR)

b) The joint distribution of the random variable (x, y) is

$$f(x,y) = \begin{cases} \frac{1}{4}[1 + xy(x^2 - y^2), for\ |x| \leq 1 \\ 0 \qquad\qquad ,for\ all\ other\ points \end{cases}$$

(i) Determine the characteristic functions of x, y, and $z = x + y$

(ii) show that the random variable x and y are dependent.

14. a) Define: Cauchy Distribution and check is it probability density function of Cauchy Distribution?

(OR)

b) Define: Laplace Distribution and find the moments of Beta Distribution.

15. a) State and prove the Bernoulli's Law of large number.

(OR)

b) State and prove the Kolmogorov inequality for large number.

PART – C

ANSWER ANY THREE QUESTIONS. **$(3 \times 10 = 30)$**

16. The density function of random variable X and Y are $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, f(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}$, and density of the joint random variable (x, y) is $f(x, y) = \frac{1}{2\pi} e^{-\frac{(x^2+y^2)}{2}}$. Derive the formulas for the distribution function and the density of the sum and product of two random variables.
17. State and prove the Lapunov inequality.
18. Define: Probability generating function and for a random variable x with probability function $P[X = k] = \binom{n}{k} p^k (1-p)^{n-k}, k = 0,1, \ldots\ldots, n$. Find probability generating function, mean, variance?
19. Define: hyper geometric distribution and let the random variable X_n, the probability function is $P[X_n = r] = \frac{n!}{r!(n-1)!} p^r (1-p)^{n-r}, r = 0,1,2, \ldots\ldots\ldots, n$, if for $n = 1,2, \ldots\ldots$, the relation $p = \frac{\lambda}{n}$, holds. $\lambda > 0$ is a constant. Prove that $\lim_{n\to\infty} P[X_n = r] = \frac{\lambda^r}{r!} e^{-\lambda}$.
20. State and prove the De Moivre – Laplace theorem.