SUB.CODE: 18UMM2C3

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DHANALAKSHMI SRINIVASAN COLLEGE
OF ARTS & SCIENCE FOR WOMEN
(AUTONOMOUS)

(For Candidates admitted from 2020-2021 onwards)

UG DEGREE EXAMINATIONS APRIL – 2021

B.SC - MATHEMATICS

TRIGONOMETRY AND VECTOR CALCULUS

Time: 3 Hrs **Max.Marks: 75**

PART– A

CHOOSE THE CORRECT ANSWER: **(10*1=10)**

1. the value of $\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \cdots \infty$
   a) cos θ b) sin θ c) cot θ d) tan θ
2. If x = cos θ + i sin θ then $x + \frac{1}{x} =?$
   a) 2 i sin θ b) 2 sin θ c) 2 i cos θ d) 2 cos θ
3. The real part of the expressions sin (x + i y)
   a) sin x cos hy b) cos x sin hy c) – sin x sin hy d) cos x cos hy
4. Sin h^{-1} x = ?
   a) $log\left(x + \sqrt{x^2+1}\right)^{-1}$ b) $\log\left(x + \sqrt{x^2-1}\right)$
   c) $log\left(x - \sqrt{x^2+1}\right)^{-1}$ d) $\log\left(x + \sqrt{x^2-1}\right)$
5. If $\nabla\phi = 2x\vec{i} - 2y\vec{j} + \vec{k}$ then the unit vector normal to the surface $x^2 - y^2 + z = 2$ at (1,-1,2) is
   a) $\frac{2\vec{i}-2\vec{j}-2\vec{k}}{3}$ b) $\frac{2\vec{i}+2\vec{j}+\vec{k}}{3}$ c) $\frac{2\vec{i}-2\vec{j}-2\vec{k}}{3}$ d) $\frac{2\vec{i}+2\vec{j}+2\vec{k}}{3}$
6. $\nabla.\left(\phi\vec{F}\right) =?$
   a) $\nabla\phi) \times \vec{F} + \phi(\nabla \times \vec{F})$ b) $(\nabla\phi).\vec{F} - \phi(\nabla.\vec{F})$
   c) $\nabla\phi).\vec{F} + \phi(\nabla.\vec{F})$ d) $\nabla\phi) \times \vec{F} - \phi(\nabla \times \vec{F})$
7. Under what condition a vector field F is conservative?
   a) curl $\vec{F} = 0$ b) div. $\vec{F} = 0$ c) div.curl $\vec{F} = 0$ d) None of these
8. The value of the integral $\int(xdy - ydx)$ around the circle $x^2 + y^2 = 1$ is
   a) $\frac{\pi}{2}$ b) π c) 2π d) 0
9. If S is any closed surface enclosed by V and $\vec{F} = x\vec{i} + y\vec{j} + z\vec{k}$ then $\iint_s \vec{F}.\hat{n}ds =?$
   a) 0 b) 3V c) 2V d) 1
10. Volume of the unit sphere is
   a) $\frac{3}{4}\pi$ b) $\frac{2}{3}\pi$ c) $\frac{3}{2}\pi$ d) $\frac{4}{3}\pi$

PART – B

ANSWER ALL THE QUESTIONS:

(5*7=35)

11. a) Prove that $(1+i\sqrt{3})^n + (1-i\sqrt{3})^n = 2^{n+1}\cos\frac{n\pi}{3}$

(OR)

b) Evaluate $\lim_{x\to 0}\frac{\sin x-\tan x}{x^3}$

12. a) If $\sin\theta = \tan hx$ prove that $\tan\theta = \sin hx$ prove also that $\cos hx = \sec\theta$

(OR)

b) Evaluate $\lim_{x\to\infty}(sinh^{-1}x - \log x)$

13. a) If $\vec{F} = x^2 + y^2\vec{j} + z^2\vec{k}$ then find $\nabla.\vec{F}$ and $\nabla\times\vec{F}$

(OR)

b) Find the directional derivative of ϕ= xyz at (1,1,1) in the direction $\vec{i}+\vec{j}+\vec{k}$

14. a) Show that $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ is a conservative vectors field.

(OR)

b) Evaluate $\iiint_s$where $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ and S is the part of the surface of the sphere $x^2+y^2+z^2=1$ lies in the first octant.

15. a) Use Divergence theorem to evaluate $\iint_s \vec{F}.\hat{n}ds$ where $\vec{F} = x^3\vec{i} + y^3\vec{j} + z^3\vec{k}$ and S is the surface of the sphere $x^2+y^2+z^2=a^2$

(OR)

b) Evaluate $\int_c\{(xy+x^2)dx + (x^2+y^2)dy\}$ where C is the curve formed by line x = 1, y = 1, x = -1, y = -1using Green's theorems

PART- C

ANSWER ANY THREE QUESTIONS:

(3*10=30)

16. If $x+\frac{1}{x} = 2\cos\theta\,, y+\frac{1}{y} = 2\cos\phi$ prove that one of the values of $\frac{x^m}{y^n}+\frac{y^n}{x^m} = 2\cos(m\theta - n\phi)$

17. If $tan\,h\,\frac{u}{2} = tan\frac{\theta}{2}$ prove that

i) $u = \log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$ ii) $\sin hu = \tan\theta$ iii) $\cos hu.\cos\theta = 1$

18. If $\phi = x^2y^3z^4$ find i) $div.\,grad\,\phi$ ii) $curl\ grad\ \phi$

19. If $\vec{F} = 2xz\vec{i} - x\vec{j} + y^2\vec{k}$ then evaluate $\iiint_v \vec{F}.dv$ where V is the region bounded by the surface x= 0, x = 2, y = 0, y = 6, $z = x^2$, z = 4.

20. Verify stoke's theorem for a vector field defined by $\vec{F} = (x^2-y^2)\vec{i} + 2xy\vec{j}$ in the rectangular region in the XOY plane bounded by the lines x = 0, x = a, y = 0, y = b.