SUB.CODE: 18UMM4C8

REG.NO: | | | | | | | | | | |

DHANALAKSHMI SRINIVASAN COLLEGE OF ARTS & SCIENCE FOR WOMEN (AUTONOMOUS)

(For Candidates admitted from 2019-2020 onwards)

UG DEGREE EXAMINATIONS APRIL - 2021

B.Sc., - MATHEMATICS

SEQUENCE AND SERIES

Time: 3 Hrs **Max.Marks: 75**

PART - A

CHOOSE THE CORRECT ANSWER **(10X1=10)**

1. The sequence 1,1,2,3,5,8,13, ... is called ---------
   a) Cauchy's sequence b) Fibonacci sequence
   c) geometric sequence d) Harmonic sequence
2. The following statements are true except-----------
   a) $\left(\frac{1}{n}\right)$is a convergent sequence b) $\left(\frac{1}{n}\right)$ is a bounded sequence
   c) $\left(\frac{1}{n}\right)$monotonic increasing d) $\left(\frac{1}{n}\right)$is a strictly monotonic decreasing
3. $\lim_{n\to\infty} \frac{n(n+1)}{n^2}$ =--------
   a) 0 b) 1 c) -1 d) ∞
4. If $(a_n) \to a$ and $a \geq 0$ for all n then-------
   a) $a_n = 0$ b) $a_n \neq 0$ c)$a_n \geq 0$ d)$a_n \leq 0$
5. If $(a_n) \to a$ and $(b_n) \to b$ then $\left(\frac{a_1 b_n + a_2 b_{n-1} + \ldots\ldots + a_n b_1}{n}\right) \to ab$ this result is known as:
   a) Cauchy's first limit theorem b) Cauchy's second limit theorem
   c) Ceasaro's theorem d) Cauchy's general principle of convergence
6. Which of the following one is wrong?
   a) $\left(\frac{1}{n}\right)$is a Cauchy's sequence b) $(n)$is not a Cauchy's sequence
   c) $\left(\frac{(-1)^n}{n}\right)$is a Cauchy's sequence d) $\left((-1)^n + \frac{1}{n}\right)$ is a Cauchy's sequence
7. Which of the following one is wrong?
   a) $1 - 1 + 1 - \cdots$oscillates finitely b) $1 + 1 + 1 + \cdots$ diverges to ∞
   c) $1 + 2 + 2^2 + \cdots$ diverges to ∞ d)$1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \cdots$ converges to $\frac{1}{2}$
8. The infinite series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if ------
   a) $p < 1$ b) $p > 1$ c)$p = 1$ d) $p \leq 1$
9. Let $\sum a_n$ be a series of positive terms. The correct statement from the following is---
   a) $\sum a_n$ converges if $\lim_{n\to\infty} \frac{a_n}{a_{n+1}} > 1$ b) $\sum a_n$ converges if $\lim_{n\to\infty} \frac{a_n}{a_{n+1}} < 1$
   c) $\sum a_n$ converges if $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} > 1$ d) $\sum a_n$ converges if $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} > 0$
10. If $a_n = \frac{2^n n!}{n^n}$then $\lim_{n\to\infty} \frac{a_n}{a_{n+1}}$ =---------
    a) $2e$ b) e c)$\frac{1}{e}$ d) $\frac{e}{2}$

PART - B

ANSWER ALL THE QUESTIONS **(5X7=35)**

11. (a)Prove that a sequence cannot converge to two different limits.

(OR)

(b) Show that the sequence $(-1)^n$ is not convergent.

12. (a) If $(a_n) \to a$ and $(b_n) \to b$ then prove that $(a_n b_n) \to ab$.

(OR)

(b) Show that $\lim_{n\to\infty} \frac{3n^2+2n+5}{6n^2+4n+7} = \frac{1}{2}$

13. (a)If a sequence (a_n) converge to l, then prove that every subsequence (a_{n_k}) of (a_n)also converge to l.

(OR)

(b)Show that (i) $\lim_{n\to\infty} \frac{1}{n}\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}\right) = 0$ (ii) $\lim_{n\to\infty} \frac{n!}{n^n} = 0$

14. (a) Let $\sum a_n$ be a convergent series converging to the sum s, then prove that $\lim_{n\to\infty} a_n = 0$. Is converse true? Justify your answer.

(OR)

(b)Discuss the convergence of$\sum \frac{1^2+2^2+ \ldots .+n^2}{n^4+1}$

15. (a) Test the convergence of $\sum \frac{n^2+1}{5^n}$.

(OR)

(b) Prove that any absolutely convergent series is convergent.

PART - C

ANSWER ANY THREE QUESTIONS **(3X10=30)**

16. Show that if (a_n) is a monotonic sequence then prove that $\left(\frac{a_1+a_2+\ . \ . \ .+a_n}{n}\right)$ is also a monotonic sequence.

17. Show that the sequence $\left(1+\frac{1}{n}\right)^n$ converges.

18. Prove that the sequence (a_n) in R is converges iff it is a Cauchy sequence.

19. State and prove Cauchy's General principle of converges of series.

20. Test the convergence of (i) $\sum \frac{n^3+a}{2^n+a}$