SUB.CODE: 18UMM6C13

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DHANALAKSHMI SRINIVASAN COLLEGE

OF ARTS & SCIENCE FOR WOMEN

(AUTONOMOUS)

(For Candidates admitted from 2018-2019 onwards)

UG DEGREE EXAMINATIONS APRIL – 2021

B.SC – MATHEMATICS

LINEAR ALGEBRA

Time: 3 Hrs **Max.Marks: 75**

PART – A

CHOOSE THE CORRECT ANSWER. **(10*1=10)**

1. Let V and W be a vector spaces over a field F. A mapping $T: V \to W$ is called a linear transformation if

   a) $T(u+v) = T(u) + T(v)$ b) $T(\alpha u) = \alpha T(u)$

   c) $T(\alpha u + \beta v) = \alpha T(u) + \beta T(v)$ d) $T(\alpha + \beta) = T(\alpha) + T(\beta)$ where$u, v \in V$ and $\alpha, \beta \in F$.

2. In $R[x]$, let $S = \{(1,0,0), (2,0,0), (3,0,0)$. Then $L(s) =$ _______

   a) S b) $\{(x, y.0) | x, y \in R\}$ c) $\{(x, 0,0) | x, y \in R\}$ d) $V_3(R)$

3. Let S be a subset of a vector space V over a field F. S is called a basis of V if

   a) S is linearly independent and $L(S) = S$

   b) S is linearly independent and $L(S) = V$

   c) S is linearly dependent and $L(S) = V$

   d) S is linearly dependent and $L(S) = S$

4. Dim $M_2(R) =$ ___

   a) 1 b) 2 c) 3 d) 4

5. The norm of the vector $(1,2,3)$ in $V_3(R)$ with standard inner product is

   a) 6 b) 14 c) $\sqrt{14}$ d) 1

6. A unit vector which is orthogonal to $(1,3,4)$ in $V_3(R)$ with standard inner product is

   a) $(1,1,-1)$ b) $(2,2,-2)$ c) $(0,4,-3)$ d) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

7. An example of a skew-symmetric matrix is ______

   a) $\begin{pmatrix} 1 & -2 & -2 \\ 2 & 1 & -3 \\ 2 & 3 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 2 & 3 & 0 \end{pmatrix}$ c) $\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$ d) $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

8. The sum of the eigen values of $\begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & -\cos\theta \end{pmatrix}$ is

a) 0 b) 1 c) $2\cos\theta$ d) $cos^2\theta$

9. The quadratic form of the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is

a) $x^2 + y^2$ b) $2xy$ c) $x^2 + 2xy$ d) $(x + y)^2$

10. Which of the following is true?

a) Any inner product on V is a bilinear form.

b) Any quadratic form can be reduced to a diagonal form.

c) Both (a) and (b)

d) None of the above

PART – B

ANSWER ALL THE QUESTIONS

(5*7=35)

11. a) Prove that, R^n is a vector space over R.

(OR)

b) Let V be a vector space over a field F. A non-empty subset W of V is a subspace of V iff $u, v \in W$and $\alpha, \beta \in F \Rightarrow \alpha u + \beta v \in W$.

12. a) Prove that, Any subset of a linearly independent set is linearly independent.

(OR)

b) Prove that, Any two bases of a finite dimensional vector space V have the same number of elements.

13. a) Find the linear transformation $T: V_3(R) \to V_3(R)$ determined by the matrix $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{pmatrix}$ w.r.t the standard basis $\{e_1, e_2, e_3\}$.

(OR)

b) State and prove Schwartz inequality.

14. a) Find the inverse of the matrix $\begin{pmatrix} 3 & 3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$ using Cayley-Hamilton theorem.

(OR)

b) Show that, Let A be a square matrix. Then the sum of eigen values of A is equal to the sum of the diagonal elements of A.Product of eigen values of $A$is $|A|$.

15. a) Let f be a bilinear form defined on $V_2(R)$ by $f(x, y) = x_1y_1 + x_2y_2$ where $x = (x_1, y_2)$and $y = (y_1, y_2)$. Find the matrix of f.

(OR)

b) Reduce the quadratic form $2x_1x_2 - x_1x_3 + x_1x_4 - x_2x_3 + x_2x_4 - 2x_3x_4$ to the diagonal form using Lagrange's method.

PART – C

ANSWER ANY THREE QUESTIONS **(3*10=30)**

16. State and Prove Fundamental theorem of homomorphism.
17. Let V be a finite dimensional vector space over a field f. Let A and B be subspaces of V. Then $\dim(A + B) = \dim A + \dim B - \dim(A \cap B)$.
18. Prove that, Every finite dimensional inner product space has an orthonormal basis.
19. Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$.
20. Show that, Let V be a vector space over a field F. Then $L(V, V, F)$ is a vector space over F under addition and scalar multiplication defined by $(f + g)(u, v) = f(u, v) + g(u, v)$and $(\alpha f)(u, v) = \alpha$