SUB.CODE: 18UMM6C14

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DHANALAKSHMI SRINIVASAN COLLEGE

OF ARTS & SCIENCE FOR WOMEN

(AUTONOMOUS)

(For Candidates admitted from 2018-2019 onwards)

UG DEGREE EXAMINATIONS APRIL – 2021

B.SC - MATHEMATICS

COMPLEX ANALYSIS

Time: 3 Hrs **Max.Marks: 75**

PART – A

CHOOSE THE CORRECT ANSWER. **(10*1=10)**

1. The Cauchy – Riemann equations can be put in the complex form as
   a) $f_x = f_y$ b) $f_x = -if_y$ c) $f_x = if_y$ d) $f_x = -if_x$
2. The analytic function in a region D with its derivative zero at every point of the domain is a
   a) Complex variable b) Continuous c) Constant d) modulus function
3. The fixed points of elementary transformation Translation is
   a) 0 b) 1 c) -1 d) ∞
4. A bilinear transformation with only one fixed point is called
   a) Parabolic b) Hyperbolic c) Elliptic d) Circle
5. Let C denotes the unit circle $|z| = 1$ then $\int_c \frac{e^z}{z} dz =$
   a) -2πi b) 2πi c) πi d) -πi
6. A bounded entire function in the complex plane is constant states
   a) Cauchy's inequality b) Liouville's theorem
   c) Morera's theorem d) Cauchy's theorem
7. Let a be an isolated singularity for f(z) then a is called……… if the principal part of f(z) at z =a has no terms
   a) removable singularity b) Poles c) essential singularity d) simple pole
8. Zero of order for the function f(z) = sinz is
   a) 0 b) 2 c) 1 d) 3
9. Residue of cot z at z = 0 is
   a) 1 d) 0 c) -1 d) 2

10. A polynomial of degree n with complex coefficients has n zeros in C states

a). Rouche's theorem
b) Argument theorem
c) Cauchy's residue theorem
d) Fundamental theorem of algebra

PART – B

ANSWER ALL THE QUESTIONS **(5*7=35)**

11. a) If $\frac{\partial^2}{\partial x \partial y} = \frac{\partial^2}{\partial y \partial x}$ prove that $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4\frac{\partial^2}{\partial z \partial \bar{z}}$

(OR)

b) Prove that $u = 2x - x^3 + 3xy^2$ is harmonic and find its harmonic conjugate.

12. a) Find the image of the circle $|z - 3i| = 3$ under the map $w = \frac{1}{z}$.

(OR)

b) Find the bilinear transformation which maps the points z = -1, 1, ∞ respectively on w = -i, -1 , i.

13. a) Show that $\int_c |z|^2 dz = -1 + i$ where C is the square with vertices O(0,0), A(1,0), B(1,1), and C(0,1).

(OR)

b)Evaluate $\int_c \frac{e^z}{(z+2)(z+1)^2} dz \text{ where } C \text{ is } |z| = 3$

14. a) Expand $f(x) = \frac{z-1}{z+1}$ as a Taylor series about z = 0 and z = 1.

(OR)

b)Suppose f(z) is analytic in a region D and is not identically zero in D then prove that set of all zeros of f(z) is isolated

15. a) Find the residues of $\frac{e^z}{z^2(z^2+9)}$ at its poles.

(OR)

b) Evaluate $\int_c \frac{e^z}{(z+2)(z-1)} dz \text{ where } C \text{ is } |z-1| = 1$

PART – C

ANSWER ANY THREE QUESTIONS **(3*10=30)**

16. State and prove Cauchy – Riemann equations.

17. Any bilinear transformation which maps the unit circle $|z| = 1$ on to the unit circle $|w| = 1$ can be written in the form of $w = e^{i\lambda}\left[\frac{z-\alpha}{\bar{\alpha}z-1}\right]$ where λ is real.

18. State and prove Cauchy's integral formula.

19. Find the Laurent's series expansions for the function $f(z) = \frac{z+4}{(z+3)(z-1)^2}$ in

(i). $0 < |z-1| < 4$ (ii). $|z-1| > 4$

20. Using Contour integration evaluate $\int_0^{2\pi} \frac{d\theta}{13+5\sin\theta}$.