SUB.CODE: 20PMM2A2

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DHANALAKSHMI SRINIVASAN COLLEGE OF ARTS & SCIENCE FOR WOMEN (AUTONOMOUS)

(For Candidates admitted from 2020-2021 onwards)

PG DEGREE EXAMINATIONS APRIL - 2021

M.Sc., - MATHEMATICS

PARTIAL DIFFERENTIAL EQUATIONS

Time: 3 Hrs **Max.Marks: 75**

PART - A

CHOOSE THE CORRECT ANSWER **(10X1=10)**

1. Eliminate the arbitrary constant a & b from z = (x+a)(y+b)

   a) z=pq   b) z=p+q   c) z =p-q   d) Z= p/q

2. The general form of first order Linear equation is

   a) $p_p - Q_q = R$ b) $p_p Q_q = R$ c) $p_p + Q_q = R$ d) $p_p / Q_q = R$

3. Along every characteristic strip of the equation F(x,y,z,p,q) =0 the function F(x,y,z,p,q) is

   a) Zero   b) constant   c) one   d) Two

4. The complete integral of the equation is pq =1

   a) $a^2x - y + az = c$ b) $a^2x - y - az = c$ c) $a^2x + y - az = c$ d) $a^2x + y - az = c$

5. The One –dimensional diffusion equation is

   a) $\frac{\partial^2 \emptyset}{\partial x} = \frac{1}{k}\frac{\partial \emptyset}{\partial t}$ b) $\frac{\partial^2 \emptyset}{\partial x^2} = \frac{1}{k}\frac{\partial \emptyset}{\partial t}$ c) $\frac{\partial^2 \emptyset}{\partial x^2} = \frac{\partial \emptyset}{\partial t}$ d) $\frac{\partial^2 \emptyset}{\partial x^2} = k\frac{\partial \emptyset}{\partial t}$

6. The general form of poissions equation is

   a) $\nabla \emptyset = 0$ b) $\nabla^2 \emptyset = 0$ c) $\nabla^2 \emptyset = 1$ d) $\nabla^2 \emptyset = 2$

7. The Characteristic function $\emptyset$ is positive then the equation is

   a) Elliptic   b) Parabolic   c) Hyperbolic   d) constant

8. The Hankel transform of K(ξ, x) is

   a) $xj_v(\xi x), v > \frac{1}{2}$ b) $xj_v(\xi x) > -\frac{1}{2}$ c) $xj_v(\xi x), v > -\frac{1}{2}$ d) $j_v(\xi x), v > -\frac{1}{2}$

9. The families of equi potential surface is

   a) f(x,y,z)=1   b) f(x,y,z)=0   c) f(x,y,z)=2   d) f(x,y,z)=c

10. The interior Neumann problem is

   a) $\frac{\partial \psi}{\partial \eta}$ Coincides   b) $\frac{\partial \psi}{\partial \xi}$ coincides   c) $\frac{\partial \theta}{\partial \eta}$ coincides   d) $\frac{\partial \psi}{\partial \alpha}$ coincides

PART- B

ANSWER ALL THE QUESTIONS **(5X7=35)**

11. a) Find the general solution of the differential equation $x^2\frac{\partial z}{\partial x} + y^2\frac{\partial z}{\partial y} = (x+y)z$

(OR)

b) Find the integral surface of the linear partial differential equation

$x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$ which contains the straight line $(x+y) = 0. z = 1$.

12. a) Find the solution of the equation $z = \frac{1}{2}(p^2+q^2) + (p-x)(q-y)$ which passes through the x-axis.

(OR)

b) Find the complete integral of the equation $p^2x + q^2y = z$

13. a) Find the solution of the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$

(OR)

b) If $\alpha_r D + \beta_r D' + \gamma_r$ is a factor of $F(D, D')$ and $\emptyset_r(\xi)$ is an arbitrary function of the single variable ξ

then if $\alpha_r \neq 0$, $u_r = \exp\left(-\frac{\gamma_r x}{\alpha_r}\right)\emptyset_r(\beta_r x - \alpha_r y)$

14. a) Solve the equation $r + 4s + t + rt - s^2 = 2$

(OR)

b) State and prove the Existence theorem in partial differential equation.

15. a) Derive the families of equi potential on the surfaces.

(OR)

b) A uniform circular wire of radius a charged with electricity of line density e surrounds grounded concentric spherical conductor of radius c. Determine the electrical charge density at any point on the conductor.

PART-C

ANSWER ANY THREE QUESTIONS **(3X10=30)**

16. If $u_i(x_1, x_2 \ldots\ldots x_n, z) = c_i$ (i=1,2,…n) are independent solutions of the equations

$\frac{dx_1}{p_1} = \frac{dx_2}{p_2} = \cdots \frac{dx_n}{p_n} = \frac{dz}{R}$ then the relation $\emptyset(u_1, u_2 \ldots u_n) = 0$ in which the function $\emptyset$ is arbitrary is a

general solution of the linear partial differential equation $p_1\frac{\partial z}{\partial m} + p_2\frac{\partial z}{\partial x_2} + \cdots p_n\frac{\partial z}{\partial x_n} = R.$

17. A necessary and sufficient condition that a surface be an integral surface of a partial differential equation s that at each point its tangent element should touch the elementary cone of the equation.

18. If $\beta_r D' + \gamma_r$ is a factor of F(D,D’) and $\emptyset_r(\xi)$ is an arbitrary function of the single variable ξ, then if

$\beta_r \neq 0, u_r = \exp\left(-\frac{\gamma_r y}{\beta_r}\right)\emptyset_r(\beta_r x)$ is a solution of the equation $F(D, D') = 0$

19. Derive the solution of the equation: $\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} = 0$ for the region $r \geqslant 0,\ z \geqslant 0$, satisfying the conditions:

$V \to 0$ as $z \to \infty$ and as $r \to \infty$

$V = f(x)$ on $z = 0,\ r \geqslant 0$

20. A rigid sphere of radius a is placed in a stream of fluid whose velocity in the undisturbed state is V. Determine the velocity of the fluid at any point of the disturbed stream.