SUB.CODE: 20PMM2C4

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DHANALAKSHMI SRINIVASAN COLLEGE OF ARTS & SCIENCE FOR WOMEN (AUTONOMOUS)

(For Candidates admitted from 2020-2021onwards)

PG DEGREE EXAMINATIONS APRIL – 2021

M.SC - MATHEMATICS

LINEAR ALGEBRA

Time: 3 Hrs **Max.Marks: 75**

PART - A

CHOOSE THE CORRECT ANSWER **(10×1 = 10)**

1. The system of equation AX=0 where A=$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then

   a) $ad-bc=0$ b) $ad-bc\neq 0$ c) $ad+bc=0$ d) $ad+bc\neq 0$

2. An $n\times n$ matrix A is called upper triangular

   a) $A_{ij}=0$ b) $A_{ji}=0$ c) $A_{ii}=0$ d) $A_{jj}=0$

3. The space V^* is denoted by

   a) L(V,V) b) L(F,F) c) L(V,F) d) L[V,F]

4. If A is an $n\times n$ matrix with entries in the field F then

   a) row rank $\neq$ column rank b) row rank = column rank

   c) row rank + column rank =2n d) None of the above

5. A polynomial f of degree n over a field has

   a) n roots b) exactly n roots c) atleast n roots d) atmost n roots

6. A linear combination of n-linear functions is

   a) n linear b) (n-1) linear c) (n+1) linear d) linear

7. An $n\times n$ matrix A over a field F is skew symmetric if

   a) $A=A'$ b) $A\equiv A'$ c) $-A=A'$ d) $A=\left(A'\right)'$

8. If $A=\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ then the characteristic polynomial is

   a) x^2-1 b) x^3-1 c) x^2+1 d) x^3+1

9. If E is a projection on R along N then (I-E) is

   a) projection on N along R b) projection on N along E

   c) projection on R along E d) is not a projection

10. If T is a linear operator on an arbitrary vector space and if there is a monic polynomial P such that

    a) P(T)=1 b) P(T)=0 c) P(T)=-1 d) P(T)=0,1

PART - B

ANSWER ALL THE QUESTIONS **(5×7 = 35)**

11. a) If A,B,C are matrices over the field F such that the products BC and A(BC) are defined prove that the products AB, (AB)C are also defined and A(BC) = (AB)C.

(OR)

b) If W is a subspace of a finite dimensional vector space V show that every linearly independent of W is finite and is part of a basis for W.

12. a) Let F be a field and T be a linear operator on F^2 defined by $T(x_1, x_2) = (x_1 + x_2, x_1)$ prove that T is non-singular.

(OR)

b) If W_1 and W_2 are subspaces of a finite dimensional vector space show that $W_1 = W_2$ if and only if $W_1^0 = W_2^0$.

13. a) State and prove Lagranges interpolation formula.

(OR)

b) Let K be a commutative ring with identity and let n be a positive integer prove that there exists atleast one determinant function on $K^{n\times n}$.

14. a) Let K be a commutative ring with identity, A and B be $n\times n$ matrices over K prove that det(AB)=(det A)(det B).

(OR)

b) Define Characteristic value, Characteristic vector, Characteristic space, Characteristic value of A in F, Characteristic polynomial.

15. a) Show that the range of T and the null space of T is invariant under T.

(OR)

b) Let F be a commuting family of diagonalizable linear operators on the finite dimensional vector space V prove that there exists an ordered basis for V such that every operator in F represented in that basis by a diagonal matrix.

PART - C

ANSWER ANY THREE QUESTIONS **(3×10 = 30)**

16. If W_1 and W_2 are finite dimensional subspaces of a vector space V prove that $W_1 + W_2$ is finite dimensional and dim W_1 +dim W_2 = dim ($W_1 \cap W_2$) + dim ($W_1 + W_2$).
17. Let V be a n-dimensional vector space over the field F and W be m-dimensional vector space over the field F show that the space L(V,W) is finite dimensional with dimension mn.
18. Let D be an n-linear function on $n\times n$ matrix over K. Suppose D has the property that D(A)=0 whenever two adjacent rows of A are equal prove that D is alternating.
19. State and prove Cayley Hamilton theorem.
20. State and prove primary decomposition theorem.