

--	--	--	--	--	--	--	--	--	--



**DHANALAKSHMI SRINIVASAN COLLEGE
OF ARTS & SCIENCE FOR WOMEN
(AUTONOMOUS)**

(For Candidates admitted from 2019-2020 onwards)

PG DEGREE EXAMINATIONS APRIL - 2021

M.Sc., - MATHEMATICS

COMPLEX ANALYSIS



Time: 3 Hrs

Max.Marks: 75

PART - A

CHOOSE THE CORRECT ANSWER

(10X1=10)

- The Intersection of a finite number of open sets is -----
 a) Closed b) open c) set d) bounded
- A metric space is said to be complete if every Cauchy sequence is -----
 a) Closed b) not convergent c) convergent d) open
- $\int_{|z|=1} e^z \cdot z^{-n} dz =$
 a) $\frac{1}{n!}$ b) $\frac{1}{(n-1)!}$ c) $\frac{2\pi i}{(n-1)!}$ d) 0
- If $f|z|$ is a polynomial and c denotes the circle $|z - a| = R$, $\int_e p(z) dz$ -----
 a) 0 b) $2\pi i$ c) $-2\pi i R^2$ d) $-2\pi i R^2 P^1(a)$
- The order of the pole $Z = 0$ for $\frac{1-\sin z}{z^5}$ is
 a) 1 b) 2 (c) 4 d) 5
- The function e^z is essential singularity at -----
 a) 1 b) 0 c) $-\infty$ d) ∞
- The value of $\int_0^{\frac{\pi}{2}} \log \sin x \, dx =$ -----
 a) $-\frac{\pi}{2} \log 2$ b) $\pi \log 2$ (c) $-\pi \log 2$ d) none of these
- The residue of $f(z) = \frac{z}{(z^2-1)^2} =$ -----
 a) 0 b) -1 c) 1 d) $-\frac{1}{2}$
- A non-constant ----- function has neither a maximum nor a minimum in its Region of definition
 a) Laplace b) Analytic c) entire d) potential
- The harmonic conjugate of $e^{x \cos y}$ is -----
 a) $e^{x \cos y} + c$ b) $e^{x \sin y} + c$ c) $e^x + c$ d) none of these

PART- B

ANSWER ALL THE QUESTIONS

(5X7=35)

11. a) Prove that a set is compact if and only if it is complete and totally bounded.

(OR)

b) Prove that on a compact set every continuous function is uniformly continuous.

12. a) State and prove Cauchy integral formula.

(OR)

b) If $f(z)$ is analytic in an open disk D , then $\int_{\gamma} f(z) dz = 0$ for every closed Curve δ in D .

13. a) A non-constant analytic function maps open sets onto open.

(OR)

b) Expand $f(z) = \frac{z-1}{z+1}$ is a Taylor's series

i. About the point $z = 0$

ii. About the point $z = 1$. Determine the region of convergent in each case.

14. a) State and Prove Argument theorem.

(OR)

b) Evaluate $\int_0^{2\pi} \frac{d\theta}{5+4 \sin \theta}$

15. a) State and Prove Schwarz's theorem.

(OR)

b) Prove that the sum of two harmonic functions is also harmonic.

PART-C

ANSWER ANY THREE QUESTIONS

(3X10=30)

16. A non-empty open set in the plane is connected if and only if any two of its points can be joined by a polygon which lies in the set.

17. State and Prove Cauchy's theorem for a Rectangle.

18. State and Prove Taylor's theorem.

19. If $f(z)$ is analytic in a region n , then $n(r, a)f(a) = \frac{1}{2\pi i} \int_{\delta} \frac{f(z)}{z-a} dz$ for every cycle δ which is homologous to zero in n .

20. If $f(z)$ is an analytic function in the annulus (or) Ring between the concentric Circles c and c' with center a and radius R and R' then $f(z) = \sum_{n=-\infty}^{\infty} A_n(z-a)^n$ Where z is any point in the ring.