SUB.CODE: 20PMM2C6

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DHANALAKSHMI SRINIVASAN COLLEGE OF ARTS & SCIENCE FOR WOMEN (AUTONOMOUS)

(For Candidates admitted from 2020-2021 onwards)

PG DEGREE EXAMINATIONS APRIL - 2021

M.Sc., - MATHEMATICS

MEASURE THEORY AND INTEGRATION

Time: 3 Hrs **Max.Marks: 75**

PART - A

CHOOSE THE CORRECT ANSWER **(10X1=10)**

1. Every non empty open set has _____________ Measure.

a) Borel b) negative c) Lebesque d) positive

2. If $P^* = [0,1] - P,\ P = \cup_{n=1}^{\infty} P_n$, then $M(P^*) = \sum_{n=1}^{\infty} \frac{2^{n-1}}{3^n} =$ ______________.

a) ∞ b) 3^{-1} c) 1 d) 0

3. Let $\{f_n,\ n = 1,2, \dots \dots \dots\}$ be a sequence of non – negative measurable functions then $\liminf \int f_n\,dx$ ___________ $\liminf \int f_n\ dx$.

a) $\leq$ b) $\geq$ c) $=$ d) $\neq$

4. The value of $\int_1^{\infty} \frac{dx}{x} =$ ___________.

a) ∞ b) $\log x$ c) 1 d) 0

5. Let A, B be subsets of a set C , let $A, B, C \in \mathbb{R}$ and let $\mu(A \cap B) =$ _________.

a) $\mu(A)$ b) $\mu(A)\,.\,\mu(B)$ c) $\mu(c)$ d) $\mu(B)$

6. Let f be a Measurable function and let $A = [x: f(x) \geq 0]$, then for $c > 0, \mu[x: f(x) > c]$_______.

a) $c\int f d\mu$ b) $-c\int f d\mu$ c) $c^{-1}\int_A f d\mu$ d) $\int_A f d\mu$

7. If $\varphi(E) = \int_E f d\mu$, where $\int f d\mu$ is defined, then φ is a ____________ Measure.

a) Jordan decomposition b) Signed c) Haln decomposition c) Borel

8. A sequence $\{f_n\}$ is fundamental in measure if for any $\in > 0, \lim_{m,n \to \infty} \mu[x: |f_n(x) - f_m(x)| > \in] =$ ___.

a) 0 b) ∞ c) 1 d) none

9. If $f(x,y) = \frac{x^2 - y^2}{(x^2+y^2)^2}, (x,y) \neq (0,0)$ then $\int_0^1 dx \int_0^1 f(x,y) dy =$ _________________.

a) $-\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $-\frac{\pi}{2}$ d) $\frac{\pi}{4}$

10. If $\{A_i\}_y$ is a monotone Sequence of sets, then $\lim A_i^y =$ ________________.

a) $\lim\,(A_i)_y$ b) $(\lim A_i)_y$ c) $(\lim A_i)^y$ d) $\lim\ A_y^i$

PART- B

ANSWER ALL THE QUESTIONS **(5X7=35)**

11. a) Show that the every interval is Measurable.

(OR)

b) Show that not every Measurable set is a Borel set.

12. a) State and prove the Lebesgue's Monotone Convergence theorem.

(OR)

b) State and prove the Lebesgue's dominated Convergence theorem.

13. a) If μ is a σ- finite Measure on a ring $\mathbb{R}$, Prove that it has a unique extension to σ- ring $S(\mathbb{R})$.

(OR)

b) Let $\{f_n\}$ be a sequence of Measurable function, $f_n: X \to [0,\infty]$; Prove that $\int \sum_{n=1}^{\infty} f_n \, d\mu = \sum_{n=1}^{\infty} \int f_n \, d\mu$

14. a) Let $f_n \to f \; a.e$, if

$(i) \mu(x) < \infty$, (ii) for each n, $|f_n| \leq g$, an integrable function, Prove that $f_n \to f \; a.u.$

(OR)

b) Let V be a signed measure on [X,S], Prove that there exists a positive set A and a negative set B, such that $A \cup B = X, A \cap B = \emptyset$, and the pair A,B is said to be a Hahn decomposition of X with respect to V. It is unique to the extent that if A_1, B_1, and A_2, B_2 are Hahn decompositions of X with respect to V. Prove that $A_1 \Delta A_2$ is a V-null set.

15. a) Let $[X, S, \mu]$ and $[Y, \mathfrak{F}, V]$, $\varphi(x) = v(V_x), \psi(y) = \mu(V^y)$, for each $x \in X, y \in Y$. Prove that φ is S-Measurable, ψ is $\mathfrak{F}$-Measurable and $\int_X \varphi \, d\mu = \int_y \psi \, dv$.

(OR)

b) If $E \in S \times \mathfrak{F}$, Prove that for each $x \in X$ and $y \in Y$, $E_x \in \mathfrak{F}$ and $E^y \in S$.

PART-C

ANSWER ANY THREE QUESTIONS **(3X10=30)**

16. Let C be any real number and let f and g be real-valued Measurable functions defined on the same Measurable set E. Prove that $f + c, cf, f + g, f - g$ and fg are also Measurable.

17. (i) If f is Riemann integrable and bounded over the finte interval [a,b], prove that f is integrable and $R\int_a^b f \, dx = \int_a^b f \, dx$.

(ii) Show that $\int_0^1 \frac{x^{\frac{1}{3}}}{1-x} log 1/x \;\; dx = 9 \sum_{n=1}^{\infty} \frac{1}{(3n-1)^2}$.

18. If f_i is Measurable, $i = 1,2, \dots \dots \dots$, prove that

(i) $\sup_{1 \leq i \leq n} f_i$ is Measurable. (ii) $\inf f_n$ is Measurable.

(iii) $\lim \inf f_n$ is Measurable. (iv) $\lim \sup f_n$ is Measurable.

19. Let V be a signed Measure on $[X, S]$, Prove that there exists Measures V^+ and V^- on $[X, S]$ such that $V = V^+ - V^-$ and $V^+ \perp V^-$. The Measures V^+ and V^- are uniquely defined by v, and $V = V^+ - V^-$ is said to be the Jordan decomposition of V.

20. State and prove Fubini's theorem.